

# GAP NOTES

KB, PM

## 1. DRAFT MANUAL FOR GRAZ GAP PACKAGE

Here an  $n$ -gon has its vertices labelled  $1, 2, \dots, n$ . An angulation  $A$  is encoded as the subset of ‘diagonal’ vertex pairs  $\{i, j\}$  that are edges of  $A$  (i.e. ignoring pairs  $\{i, i + 1\}$ , which are always in, as edges of the  $n$ -gon itself).

### 1.1. Functions defined in GRaZ.

`Baur(n,m)` returns the list of angulations of the  $n$ -gon with  $m$  diagonals. Thus

```
gap> Baur(5,1);
[[ [ 1, 3 ] ], [ [ 1, 4 ] ], [ [ 2, 4 ] ], [ [ 2, 5 ] ], [ [ 3, 5 ] ] ]
```

`Baurtab(n,m)` returns for each angulation  $A$  from `Baur(n,m)`:

1. a sequence  $tab = [r_1, r_2, \dots]$  where  $r_1$  is (the size of) an ear tile of  $A$ ;  $r_2$  is an ear tile of  $A/r_1$  and so on. Thus  $tab$  gives the sizes of all the tiles of  $A$  (but in no particular order).
2. a list giving the multiplicities of the various tile sizes — the partition associated to  $A$ .
3. a running count of the number of times various specific partitions occur. (Ignore this! See below.)
4. a running total count of the angulations. Thus

```
gap> Baurtab(5,1);
[ 3, 4 ] [ 0, 0, 1, 1, 0, 0, 0, 0, 0, 0 ] [ 0, 0, 0 ] 1
[ 4, 3 ] [ 0, 0, 1, 1, 0, 0, 0, 0, 0, 0 ] [ 0, 0, 0 ] 2
[ 3, 4 ] [ 0, 0, 1, 1, 0, 0, 0, 0, 0, 0 ] [ 0, 0, 0 ] 3
[ 4, 3 ] [ 0, 0, 1, 1, 0, 0, 0, 0, 0, 0 ] [ 0, 0, 0 ] 4
[ 3, 4 ] [ 0, 0, 1, 1, 0, 0, 0, 0, 0, 0 ] [ 0, 0, 0 ] 5
```

`Baurtabb(n,m,reftab)` is as above but in 3. counts occurrences of the partition `reftab`, given in the ‘exponent’ format  $[t_1, t_2, \dots, t_{10}]$ . (I.e.  $t_r$  is the number of  $r$ -gonal tiles. Thus this argument should always start  $[0, 0, t_3, \dots]$ . Note that the array size 10 is coded in, but is arbitrary and needs to be fixed for larger  $n$ !)

1.2. **Jobs.** Main: Write the code to compute the Scott permutation of each angulation.

#### REFERENCES

- [1] K. Baur, A. King, R.J. Marsh, *Dimer models and cluster categories for Grassmannians*, preprint, 1309.6524v1.pdf
- [2] T.K. Petersen, P. Pylyavskyy, B. Rhoades, *Promotion and cyclic sieving via webs*. J. Algebraic Combin. 30 (2009), no. 1, 19–41
- [3] J.H. Przytycki, A.S. Sikora, *Polygon dissections and Euler, Fuss, Kirkman and Cayley numbers*, preprint, arXiv:math/9811086v1