## CODING THEORY: PROBLEMS 1

Here we may write $\Sigma_m$ for the set of $m$ symbols $\{0, 1, 2, ...\}$. For example $\Sigma_3 = \{0, 1, 2\}$.

(1) Write out all the elements of the set $\{1, 2\}^3$, explaining any notation you use.

(2) For each of the following codes $C_i \subset \Sigma_3^3$, $i = 1, 2, \ldots, 5$, calculate the minimum distance $d(C_i)$.

  (a) $C_1 = \{000, 111\}$,

  (b) $C_2 = C_1 \cup \{222\}$,

  (c) $C_3 = C_2 \cup \{012\}$,

  (d) $C_4 = C_3 \cup \{011\}$,

  (e) $C_5 = C_4 \cup \{210\}$.

(3) For each of the following codes $C_i \subset \Sigma_2^4$, $i = 1, 2, 3$, calculate $d(C_i)$.

  (a) $C_1 = \{0000, 0111\}$,

  (b) $C_2 = \{0000, 0111, 1110\}$,

  (c) $C_3 = \{0000, 0111, 1110, 0101\}$.

(4) Given a code $C \subset \Sigma_q^n$ what is the smallest possible value of $d(C)$ if:

  (a) $C$ is 7 error correcting,

  (b) $C$ is 11 error detecting,

  (c) $C$ is 11 error correcting,

  (d) $C$ is 21 error detecting.

(5) Let $C$ be a binary $(9, 6, 5)$ - code transmitted over a binary symmetric channel with symbol error probability $p = 0.01$. Find a good upper bound on the word error probability for any codeword (i.e. find a good upper bound on the probability that a sent code word will be incorrectly decoded).

(6) If possible, construct a binary $(n, M, d)$-code with each of the following parameters:

$$(9, 2, 9),\ (3, 8, 1),\ (4, 8, 2),\ (5, 3, 4),\ (8, 41, 3)$$

In each case if no such code exists, prove it.

(7) In the table of known values for $A_2(n, d)$ in the notes (page 42) there are four pairs $(n, d)$ where $max(M) = A_2(n, d)$ is the same as the BP-bound (marked with a $*$ in the table). Which, if any of these pairs correspond to perfect codes?