CODING THEORY: PROBLEMS 2

Recall that $\Sigma_3 = \{0, 1, 2\}$ and so on.

(1) Suppose $C \in \Sigma_3^6$. Write down the weights of the following codewords $x_i \in C$, $i = 1, ..., 4$.
   (a) $x_1 = 000000$
   (b) $x_2 = 000100$
   (c) $x_3 = 002120$
   (d) $x_4 = 212121$

(2) In the field $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ calculate:
   (a) $4^{44} + 3^{23}$,
   (b) $4^{44} - 3^{23}$,
   (c) $4^{88} \times 3^{46}$,
   (d) $4^{45} \div 3^{26}$.

(3) Construct a table of multiplicative inverses for $\mathbb{Z}_7$, and for $\mathbb{Z}_{17}$.

(4) Do there exist finite fields of the following orders?
   (a) 19?
   (b) 200?
   (c) 625?
   (d) 1026?

(5) A polynomial (in variable x, say) *over a field* F is a polynomial with coefficients taken from the field F. Let $p(x)$ be a polynomial over F. A *root* of $p(x)$ is an element $a \in F$ such that $p(a) = 0$. A quadratic (or cubic) polynomial over F is said to be *irreducible* if it has no roots in F.
   (a) Write down all degree 2 polynomials over the field $\mathbb{Z}_2$.
   (b) Which if any of these polynomials is irreducible?

(6) Although $\mathbb{Z}_4$ is not a field, we can still find a field of order 4. Construct addition and multiplication tables for a set $F_4 = \{0, 1, a, b\}$ (say), such that F_4 becomes a field, where 0 is the additive identity and 1 is the multiplicative identity.