# CODING THEORY: PROBLEMS 3

(1) Let $C$ be a subspace of $\mathbb{Z}_3^5$ having $\{(1,2,0,0,1),(2,0,1,2,1),(0,2,1,2,1)\}$ as a spanning set. Find a basis for $C$. What is $\dim(C)$?

(2) Consider the following sets:

- $S_1 = \{0000, 0001, 1000\} \subset \mathbb{Z}_2^4$;
- $S_2 = \{0000, 0001, 1000, 1001\} \subset \mathbb{Z}_2^4$;
- $S_3 = \{00000, 01110, 01000, 00110\} \subset \mathbb{Z}_2^5$;
- $S_4 = \{0000, 0001, 1000\} \subset \mathbb{Z}_3^4$;
- $S_5 = \{0000, 0001, 0002\} \subset \mathbb{Z}_3^4$;

Determine which of these are linear codes (giving reasons for your answers).

(3) For each of the $S_i$ in Question 2 which *are linear codes* , write down a generator matrix for the code.

(4) Let $C_i$ be a $3-ary$ linear code generated by $G_i$ where:

$$G_1 = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix} \quad G_2 = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

For each $i = 1, 2$, what is $|C_i|$? List the codewords of $C_i$ and hence compute its minimum distance.

(5) Let $C$ be the binary linear code generated by:

$$G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Find a generator matrix for the same code $C$ in standard form.

(6) For each of the following linear codes $C_i \subset \mathbb{Z}_3^4$, $i = 1, 2, 3$, calculate $w(C_i)$.

(a) $C_1 = \{0000, 0111, 0222\}$,

(b) $C_2 = \{0000, 0100, 0200\}$,

(c) $C_3 = \{0000, 0120, 0210\}$.

(7) Prove that $w(C) = d(C)$ for any linear code $C$.