CODING THEORY: PROBLEMS 4

(1) Let C_i, $i = 1, 2$ be the binary linear codes generated by,

$$G_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \; G_2 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(a) Construct the standard array for C_1 and C_2.

(b) Using C_2,

(i) Decode the received vectors 1101 and 0011.

(ii) Give an example of two errors occurring and being corrected.

(iii) Give an example of two errors occurring and not being corrected.

(c) Assume code C_2 is transmitted down a binary symmetric channel with symbol error probability $p = 0.01$. As this is a linear code the word error probability is independent of the codeword transmitted. Compute $P_{err}(C_2)$.

(d) If C_2 is only used for error detection, what is the probability $P_{undetec}(C_2)$ that transmitted codeword will be received with undetected errors, under the assumption of part b)?

(2) Let C be be the $3-ary$ $[4, 3]$ code generated by,

$$G = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Find a parity check matrix for C and hence list the codewords of the dual $C^{\perp}$.

(3) Let $C \subset \mathbb{Z}_7^5$ be the linear code with generator matrix,

$$G = \begin{pmatrix} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 5 & 6 \end{pmatrix}$$

(a) Write down a parity check matrix H for C.

(b) Compute the matrix $G \cdot H^t$. What do your results tell you?

(c) Show that $d(C) = 3$.

(d) Find how many coset leaders there are and how many have weight 1.

(e) Decode the received vector 11254 where 1 error has occurred in transmitting.