CODING THEORY: PROBLEMS 5

(1) The 26 letters of the alphabet may be represented in $\mathbb{Z}_3^3$ by $A \mapsto 001$, $B \mapsto 002$, $C \mapsto 010$, ..., $Z \mapsto 222$. Let us also represent space by 000. We are given the parity check matrix,

$$H = \begin{pmatrix} 1 & 0 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

of a 3−ary $[6, 3, d]$-code C, i.e. $w \in C$ if and only if $H.w^t = 0$.

(a) Note that H is not in standard form. Confirm that

$$G = \begin{pmatrix} 2 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 2 & 1 \end{pmatrix}$$

is a generator matrix for C.

(b) G may be used to encode elements $u = (u_1, u_2, u_3)$ of $\mathbb{Z}_3^3$ by $u \mapsto x = uG$. It may therefore be used to encode letters of the alphabet, via the above representation. Compute the encoded form of the letter P.

(c) You are now told that $d(C) = 3$. Compute the syndromes for all the coset leaders of weight 1.

(d) Decode as much as possible of the following received message, given that the transmitted message was encoded using C with generator matrix G, assuming nearest neighbour decoding.

Message:

212012	012212	220112	112100	220112	000000
200021	112000	220112	000000	022021	221000
022200	002000	022021	202100	111112	012022

(2) Determine which of the following codes $C_i, i = 1, .., 4$ are cyclic, explaining your answer.

(a) The binary code $C_1 = \{00000, 10110, 01101, 11011\}$,

(b) The ternary code $C_2 = \{0000, 1212, 2121\}$,

(c) The binary code $C_3 = \{000, 010, 100, 001\}$,

(d) The binary code $C_4 = \{0000, 1010, 0101, 1111\}$.

(3) Write out the parity check and generator matrices for the following hamming codes.

(a) $Ham(\mathbb{Z}_2^4)$

(b) $Ham(\mathbb{Z}_7^2)$