Integer Partitions

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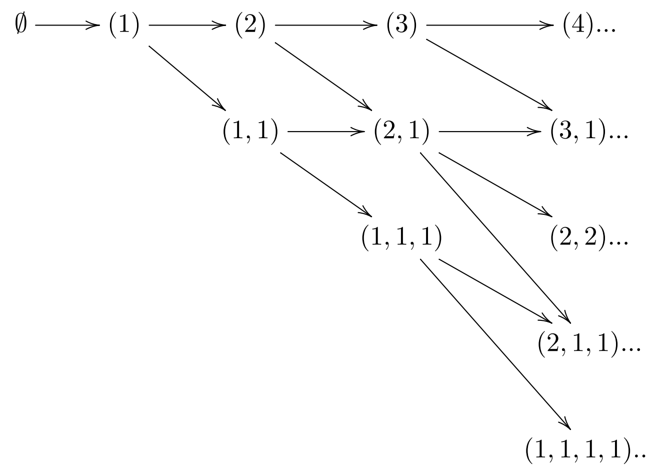
An integer partition λ of integer $n \in \mathbb{N}$ is a sequence $\lambda = (\lambda_1, \lambda_2, \lambda_3, ...)$ with terms $\lambda_i \in \mathbb{N}_0$, such that $\lambda_i \geq \lambda_{i+1}$, and the sum of all terms $\Sigma_i \lambda_i = n$.
For example $(3, 2, 1, 1)$ is a partition of 7. It can be represented pictorially as

$$(3,2,1,1) \quad = \quad \begin{array}{lll} \star & \star & \star \\ \star & \star & \\ \star & & \\ \star & & \end{array}$$

Let us write Λ for the set of all integer partitions. For $\lambda \in \Lambda$ we write $\lambda \vdash n$ or $|\lambda| = n$ if λ a partition of n. For example $(4, 2, 1, 1) \vdash 8$. Two partitions λ, μ are 'adjacent' in Λ if $\lambda_i = \mu_i$ for all i except one term, where $\lambda_i = \mu_i \pm 1$. For example $(4, 2, 1, 1)$ and $(3, 2, 1, 1)$ are adjacent. We write Y_Λ for the infinite graph with vertex set Λ and edges given by adjacency (we can direct these edges from the smaller to the larger partition):



This project aims to explore, understand and explain some of the many interesting properties of the set Λ and its adjacency graph Y_Λ.

The set Λ has some very interesting connections, for example:

- to physics and geometry through various routes - for example questions about the expected shape of the corner of a crystal;
- to algebra and groups through the symmetric groups; and
- to combinatorics through various routes - for example properties of directed walks on the graph Y_Λ (that is, sequences of partitions where the successor λ' to λ is adjacent but bigger).

Your project would focus on one of these aspects, with the aim of explaining this open world and understanding some part of it; and the objective of explaining to an interested non-expert.

(This project can have a small; medium; or big computational component, as suits you. There are no prerequisites outside the core modules.)