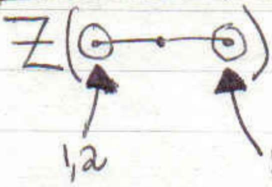
Stat. Mech. 4 Paul Martin

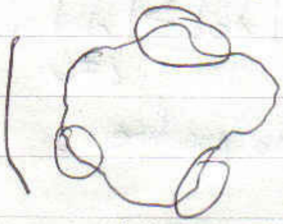
30/10/08 9

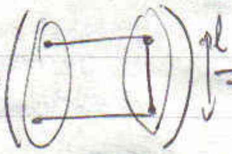
Z() $= \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & x & 1 \\ 2 & 1 & x \end{array}^{\,2} = \begin{pmatrix} x & 1 \\ 1 & x \end{pmatrix}\begin{pmatrix} x & 1 \\ 1 & x \end{pmatrix} = \begin{pmatrix} x^2+1 & 2x \\ 2x & x^2+1 \end{pmatrix}$

Correlation functions
– type of observable that will appear.
Algebraic representation
Percolations.

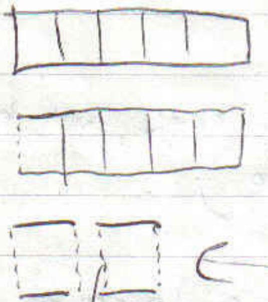
Z() $= \begin{pmatrix} x & 1 \\ 1 & x \end{pmatrix}$ ← Transfer matrix, typically square.

Canapolic.

Z()

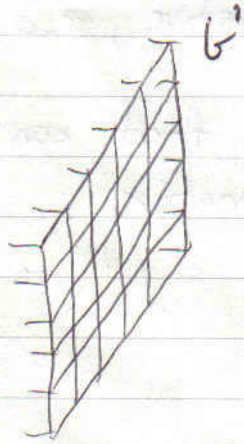
Z() $= \begin{array}{c|cccc} & 11 & 12 & 21 & 22 \\ \hline 11 & x^3 & x & x & x \\ 12 & x^2 & x^2 & 1 & x^2 \\ 21 & x^2 & 1 & x^2 & x^2 \\ 22 & x & x & x & x^3 \end{array}$

$\boxed{m = l}$



TL algebra.

$\frac{1}{2}$ the interaction in each block
– combine to make the whole.



$G' \times A_L$.

subalg. of partition algebra.

Eigenvalues of T. $Tv_i = \lambda v_i$.

Symmetrizing here corresponds to ~~symmetrizing~~ similarity transformation.

So T is (similar to) a real symmetric matrix.

For now, join front to back (this choice doesn't affect the physics very much)

$$Z_n^{p\text{-periodic}} = \mathrm{Tr}\, T^n = \sum_i \lambda_i^n$$

(n = # layers.)

Identify LH edge with RH edge → so theorem from last week gives trace.

Perron Frobenius Theorem (PF): for nonnegative T $\exists\ |\lambda_0| > |\lambda_i|$, $i \neq 0$. λ_0 positive.

(Note that $x > 0$ as temperature is real).

So $\frac{1}{nm} \ln Z = \frac{1}{nm} \ln\left(\lambda_0^n \left(1 + \sum_{i \neq 0} \left(\frac{\lambda_i}{\lambda_0}\right)^n\right)\right)$.

N = total no vertices
n = # layers.
~~...~~
$l = m$.

becomes $\lim_{N \to \infty} f_N = \frac{1}{m} \ln \lambda_0$.
(λ_0 depends on m.)

(Note: here m is fixed, but in the 2-d Ising model case, we have to deal with $m \to \infty$ as well. Not considered here).

NB: In this setting, lim makes sense, since G is n layers of m-site wide: n varying, m fixed.

PF also ~~says~~ says: v_0 (λ_0-eigenvector) can be taken to be positive, so

$(T^n)_{ij}$, $\sum_{ij} (T^n)_{ij}$, $\mathrm{Tr}\, T^n$ are all scalars that can be extracted from the matrix.

$T^n e$ (elementary matrix) also real nonnegative, so not orthogonal to v_0.

So we'll get the same answer as above in the limit.

All broadly "analytically" the same thing, all these polynomials are approximations to λ_0^n ... which has a lot more analytic structure. (Here it is stable.)