THE CENTRE OF QUANTUM $\mathfrak{sl}_n$ AT A ROOT OF UNITY

Let $\mathfrak{g} = \mathrm{Lie}(G)$ be the Lie algebra of a reductive group G over an algebraically closed field K. As is well known, the universal enveloping algebra $U(\mathfrak{g})$ of $\mathfrak{g}$ has no zero divisors, so its centre Z is an integral domain. If K is of characteristic zero, then it is known that Z is a polynomial algebra in $\mathrm{rank}(G)$ variables.

Now assume that K is a of characteristic $p > 0$. Then there is a subalgebra Z_p of Z which is a polynomial algebra in $\dim(\mathfrak{g})$ variables. Furthermore Z and U are finite modules over Z_p. In particular Z has Krull dimension $\dim(\mathfrak{g})$. The spectrum $\mathrm{Maxspec}(Z)$ of Z is no longer a smooth variety. However, it is conjectured that Z is a unique factorization domain and that it has a purely transcendental field of fractions.

For quantized enveloping algebras associated to a complex finite dimensional simple Lie algebra $\mathfrak{g}$, very similar phenomena occur. The quantized enveloping algebra $U_q(\mathfrak{g})$ at a transcendental parameter q plays the rôle of the universal enveloping algebra in characteristic 0. The specialization $U_\varepsilon(\mathfrak{g})$ of $U_q(\mathfrak{g})$ at a primitive l-th root of unity ε, as defined by De Concini, Kac and Procesi, plays the rôle of the universal enveloping algebra in positive characteristic.

In my talk I will discuss the results of my work on the above conjectures for the centre of $U_\varepsilon(\mathfrak{sl}_n)$. I will also indicate the relevance of the study of the centres of $U(\mathfrak{g})$ (in positive characteristic) and $U_\varepsilon(\mathfrak{g})$ for the representation theory of $\mathfrak{g}$ and $U_\varepsilon(\mathfrak{g})$ and I will mention the relation with the Gelfand-Kirillov conjecture.